Name____________________

MTH 287– HW #3

(due Monday July 18, 2014)

Please write all of your answers on your own paper, numbered down the LEFT side in order. Problems done on the right side or out of order may receive no credit.

Prove by Induction:

1) $2^2+4^2+6^2+\ldots+(2n)^2=\dfrac{2n(n+1)(2n+1)}{3}$ for $n \geq 2$

2) $3 \mid (n^3+2n)$ for integers $n \geq 0$

3) If one had an infinite supply of 4-cent and 7-cent stamps then every postage ≥ ______ could be made.

Bonus A) $n! > 2^n$ for integers $n \geq 4$

4) Use the Binomial Theorem and Pascal's Triangle to simplify: $(x-2)^6$

5) a) Prove that $f(x)=\dfrac{2x-3}{5x+7}$ is one-to-one

b) find its inverse function $f^{-1}(x)$

Bonus B) How many functions are there of the form: $(x)=mx+b$, where m and b are real numbers? How do you know?

For each relation, determine if it is (a) Reflexive (b)Symmetric (c) Antisymmetric (d)Transitive. If it has the property, show how you know, if it does not, give a counterexample.
(e) Is it an equivalence relation? If so, give examples of some members of its equivalence class.

6) A = the set of all Real numbers. R = $\{ (x, y) \mid x^2 \geq x+y \}$

7) A = the set of all positive Integers. R = $\{ (x, y) \mid x$ or y is prime$\}$

8) A = the set of Real numbers. R = $\{ (x,y) \mid xy \geq 0\}$

For each relation, determine if it is (a) Reflexive (b)Symmetric (c) Antisymmetric (d)Transitive.
If it has the property, show how you know, if it does not, give a counterexample.
(e) Is it an equivalence relation? If so, give examples of some members of its equivalence class.

9) A = the set of positive integers.
$R = \{ (x,y) \mid x \equiv y \pmod{3} \}$

10) A = the set of ordered pairs in the plane.
$R = \{ ((w,x), (y,z)) \mid w \geq y \text{ or } x \geq z \}$

Bonus C) A = set of all ordered triples of integers.
R = { ((x,y,z), (a,b,c)) | x and a have at least 2 factors in common, OR y and b have at least 2 factors in common, OR z and c have at least 2 factors in common }